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Renormalization of Currents for Massive Fermions* FERMILAB-CONF-98/332-T

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The renormalization of vector and axial-vector currents for massive fermions (in the “Fermilab formalism”) is discussed. We give results for non-degenerate masses, which are needed for semi-leptonic form factors.

1. INTRODUCTION

Like many groups, we and our collaborators are calculating form factors of semi-leptonic decays,

$$B \rightarrow D l \nu \text{ [1]}, \quad B \rightarrow \pi l \nu \text{ [2]}, \quad (1)$$

$$D \rightarrow K l \nu \text{ [3]}, \quad D \rightarrow \pi l \nu \text{ [3]}, \quad (2)$$

and similar decays to the vector mesons D^* , K^* , or ρ . This paper discusses the renormalization—including effects of the nonzero quark masses—of the weak currents inducing the transitions. The form factors are needed to an accuracy of a few per cent, so it would be ideal to devise a fully non-perturbative renormalization program. We show here how certain ratios of transitions induced by vector and axial-vector currents take care of most, but not all, of the renormalization nonperturbatively. We also give the residual one-loop radiative corrections to these ratios. They are a few per cent. One should expect, therefore, that the uncalculated higher-order corrections will not be needed for some time.

Our analysis differs from that of the *Alpha* Collaboration [4] in its treatment of the quark masses. First, in no decay may the quarks be considered degenerate. Second, the masses of the charmed and bottom quarks are larger than the energy scale of chromodynamics, Λ_{QCD} . Consequently, chiral Ward identities are not helpful for normalizing axial-vector currents. Third, these masses are usually, in practice, not small compared to the lattice cutoff. Therefore, to isolate the leading mass dependence into coefficient functions (rather than matrix elements) we apply the mass-dependent improvement program of

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Ref. [5] to the clover action. In particular, we keep nonzero quark masses in Feynman diagrams; for a transition such as $b \rightarrow u$ the up quark’s mass can always be set to zero later.

At tree level currents suitable for massive fermions are given by [5,6]

$$\mathcal{V}_\mu^{cb} = Z_{Vcb} \bar{\Psi}^c \gamma_\mu \Psi^b \quad (3)$$

$$\mathcal{A}_\mu^{cb} = Z_{Ac b} \bar{\Psi}^c \gamma_\mu \gamma_5 \Psi^b \quad (4)$$

where, with κ denoting the hopping parameter,

$$\begin{aligned} \Psi^f &= \sqrt{2\kappa^f} \left(1 + a d_1^f \gamma \cdot \mathbf{D} \right) \psi^f, \\ \bar{\Psi}^f &= \sqrt{2\kappa^f} \left(\bar{\psi}^f - a d_1^f (\mathbf{D} \bar{\psi}^f) \cdot \gamma \right), \end{aligned} \quad (5)$$

and ψ^f is the field of flavor f in the hopping-parameter form of the action. In the following, we speak of charmed and bottom quarks, but c and b really stand for distinct quark flavors.

The renormalization factors Z depend on both masses in the current. At tree level the mass dependence factorizes

$$Z_{Vcb}^{[0]} = Z_{Ac b}^{[0]} = e^{M_1^c/2} e^{M_1^b/2}, \quad (6)$$

where M_1^f is the rest mass of flavor f . Beyond tree level $Z_V \neq Z_A$ and the mass dependence no longer factorizes. The main aim of this paper is to obtain the full mass dependence of Z_V and Z_A in one-loop perturbation theory. Note that for degenerate quarks the vector current’s absolute normalization can be used to define Z_{Vff} non-perturbatively.

The coefficient d_1^f depends on the mass of flavor f . Beyond tree level there is no reason to expect that a universal rotation à la Eq. (5) improves all currents. One strategy is to define d_1^f

by requiring that the equal-mass vector current be conserved. For other currents and when the masses are not the same, further improvement is attained by adding higher-dimension terms— $\partial_j \bar{\Psi} i \sigma_{ij} \Psi$, $\bar{\Psi} D_i \Psi$, and $\partial_i \bar{\Psi} \gamma_5 \Psi$ —to the right-hand sides of Eqs. (3) and (4), as is done with very light quarks [4] and in nonrelativistic QCD [7,8].

2. VECTOR CURRENT

Most of the renormalization can be captured by writing

$$Z_{Vcb} = Z_{Vcc}^{1/2} Z_{Vbb}^{1/2} R_{Vcb}. \quad (7)$$

The equal-mass factors Z_{Vqq} are obtained non-perturbatively, leaving the expansion

$$R_{Vcb} = 1 + \sum_{l=1}^{\infty} g_0^{2l} R_{Vcb}^{[l]}. \quad (8)$$

The one-loop term $R_{Vcb}^{[1]}$ is set by requiring

$$\frac{\langle c, \xi' | \mathcal{V}_0^{cb} | b, \xi \rangle}{\bar{u}_{c, \xi'} \gamma_0 u_{b, \xi}} = 1 + g^2 r^{[1]}, \quad (9)$$

to one loop, where

$$r^{[1]} = \frac{3C_F}{16\pi^2} \left(\frac{m_b + m_c}{m_b - m_c} \ln \frac{m_b}{m_c} - 2 \right). \quad (10)$$

The only Feynman diagram needed to compute $R_{Vcb}^{[1]}$ is the vertex diagram. Everything associated with the legs—self energies and the factors $e^{M_1^b/2} e^{M_1^c/2}$ —drop out by construction. From the vertex diagram with unequal masses one must subtract the average of the diagram with equal masses. From this combination one also must subtract the corresponding combination of the continuum vertex diagram. This gives $-R_{Vcb}^{[1]}$. The resulting loop integral is ultraviolet and infrared convergent. (Infrared cancellation occurs point-by-point if the masses in continuum propagators are taken equal to the corresponding kinetic masses.)

Before showing results it is worthwhile to anticipate the outcome. See Fig. 1. R_{Vcb} is symmetric under interchange of b and c . When the two masses are equal $R_{Vcb}^{[l]} = 0$, $l \geq 1$, by construction. When both masses are far below the

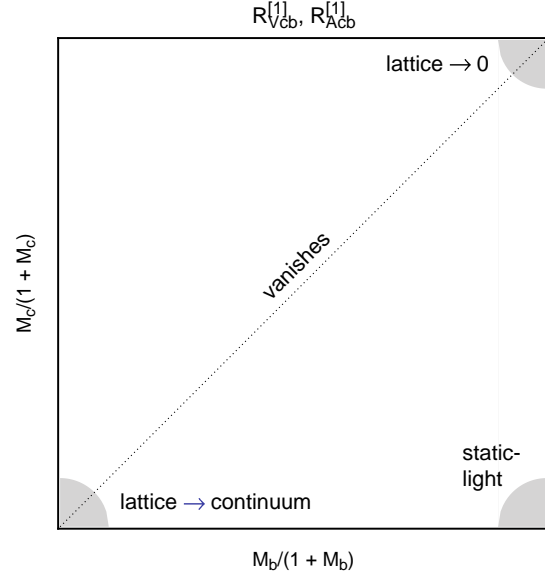


Figure 1. Limiting behaviors of $R_{Vcb}^{[l]}$, $R_{Acb}^{[l]}$.

lattice cutoff, $R_{Vcb}^{[1]}$ must vanish as $m_{b,c}a$ to a power. (The linear term should be proportional to $1 - c_{\text{SW}}$, and it is.) When both masses are much larger than the cutoff, the heavy-quark flavor symmetry (of the lattice action) ensures that the lattice contribution vanishes, i.e., $R_{Vcb}^{[1]} \rightarrow r^{[1]}$. When one mass is larger than the cutoff and the other smaller, $R_{Vcb}^{[1]}$ should resemble the radiative correction of a “static-light” current: dependence on the light mass should drop out, leaving a logarithm $(3C_F/16\pi^2) \ln M_2^b a$ plus a constant [9].

Fig. 2 bears out these expectations for a set of masses with $m_c = 0.256m_b$, appropriate to charmed and bottom quarks. These results have been obtained independently by the two authors.

To illustrate how to apply these results, let us consider the quark masses relevant to our calculation of the semileptonic decay $B \rightarrow D l \nu$ at $\beta = 5.7$ (and $\tilde{c}_{\text{SW}} = 1$), for which $m_b a = 3.9$ and $m_c a = 1.0$ [1]. Reading off Fig. 2 one finds $R_{Vcb}^{[1]} = 0.0076$. For the strong coupling we consider the range $\alpha_s(\pi/a) = 0.19$ to $\alpha_s(1/a) = 0.33$. Thus, we multiply the bare matrix element with

$$R_{Vcb} = 1 + 0.0076 \times 4\pi \times \alpha_s = 1.025(6). \quad (11)$$

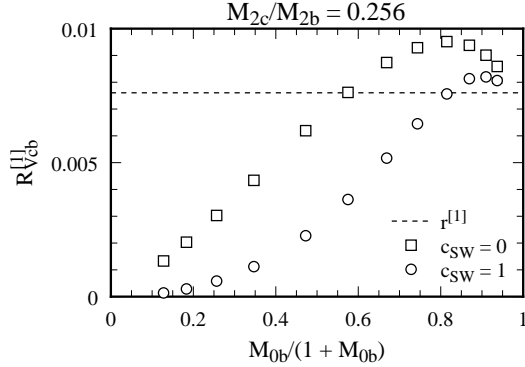


Figure 2. Radiative corrections to the $b \rightarrow c$ vector current. Circles (squares) denote $c_{\text{SW}} = 1$ (0).

This estimate will be refined when we complete the calculation of the BLM [10] matching scale q^* .

3. AXIAL-VECTOR CURRENT

For the axial-vector current one could compute $Z_{A^{cb}}/Z_{V^{cb}}$, with the denominator from Eq. (7). For our analysis of $B \rightarrow D^* l \nu$, however, we need the ratio

$$R_{A^{cb}} = \frac{Z_{A^{cb}}}{Z_{A^{cc}}^{1/2} Z_{A^{bb}}^{1/2}}. \quad (12)$$

The one-loop term $R_{A^{cb}}^{[1]}$ is set by analogy with Eq. (9), replacing \mathcal{V}_0 and γ_0 with \mathcal{A}_i and $\gamma_i \gamma_5$, but $r^{[1]}$ remains as in Eq. (10). The limiting behaviors of $R_{A^{cb}}^{[1]}$ are as for $R_{V^{cb}}^{[1]}$. Our results are in Fig. 3.

4. OTHER COMPONENTS

Scattering matrix elements of \mathcal{V}_i and \mathcal{A}_0 vanish as the three-momenta go to zero. They can take the same renormalization factors as \mathcal{V}_i and \mathcal{A}_0 , but commensurate effects are the one-loop corrections to d_1 and to the coefficients of other higher-dimension improvements. The nonperturbative construction of a normalized, conserved \mathcal{V}_μ^{ff} implies that perturbation theory is needed only for “ $m_b - m_c$ ” and “ $V - A$ ”.

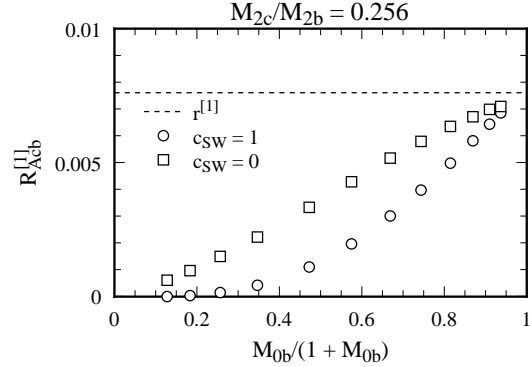


Figure 3. Radiative corrections to the $b \rightarrow c$ axial-vector current. Symbols as in Fig. 2.

5. SUMMARY

By computing most of the renormalization non-perturbatively, it is possible to reduce the one-loop corrections. This is achieved by requiring a correctly normalized, conserved vector current, or by extracting physics from a ratio of correlators, such that Eq. (12) applies. Indeed, the residual coefficients shown in Figs. 2 and 3 are small. Hence, the uncalculated higher-order radiative corrections to semi-leptonic form factors are unlikely to be large.

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